Shear Viscosity for a Debye-Hückel Plasma

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The Kubo formula for the shear viscosity of a Debye-Hückel plasma is evaluated by an extension of ERNST's method. In the main the result can be shown to be equivalent to the result derived by BRAUN from the LENARD-BALESCU equation with the CHAPMAN-ENSKOG method. Moreover a correction of BRAUN's result can be given as caused by the particle interaction.

1. Introduction

In classical statistics transport coefficients for special systems may be obtained from the Liouville equation in two different ways. On the one hand, kinetic equations for special systems which are solved in the hydrodynamic approximation for small deviations from equilibrium, are derived by corresponding approximations. With the solutions of these linearized equations the transport coefficients are calculated. On the other hand, general expressions for transport coefficients, so-called Kubo formulae, are derived from the Liouville equation by linearization (vid. e. g. [1]). The transport coefficients are represented by equilibrium averages of microscopic time correlation functions. The Kubo formulae have to be evaluated for special systems.

For a plasma in Debye-Hückel approximation, the LENARD-BALESCU equation [2] is valid as a kinetic equation for the one-particle distribution function $n_{1,t}$. This equation can be derived by perturbation expansion and partial summation assuming the mean effective strength of the internal interaction to be much smaller than the mean thermal energy and assuming further the effective range of the interaction to be the Debye length [3]. In [3] it took the form

$$i \frac{\partial}{\partial t} n_{1,t} = \boldsymbol{l}_1 n_{1,t} + \int \mathrm{d}2\, \boldsymbol{l}_{12}\, n_{1,t}\, n_{2,t} + i \int \mathrm{d}\mathfrak{k}\, \mathrm{d}\mathfrak{p}_2\, \mathfrak{k} \left(\frac{\partial}{\partial \mathfrak{p}_1} - \frac{\partial}{\partial \mathfrak{p}_2}\right) \left|\frac{u_k}{\varepsilon_{1,\mathfrak{k}}}\right|^2 \delta\left(\frac{\mathfrak{p}_1 - \mathfrak{p}_2}{m}\, \mathfrak{k}\right) \mathfrak{k} \left(\frac{\partial}{\partial \mathfrak{p}_1} - \frac{\partial}{\partial \mathfrak{p}_2}\right) n_{1,t}\, n_{2',t} \tag{1}$$

with the abbreviations $\boldsymbol{l}_1 \equiv -i(\mathfrak{p}_1/m)\, \partial/\partial \mathfrak{r}_1$, $\quad \boldsymbol{l}_{12} \equiv i \frac{\partial u_{12}}{\partial \mathfrak{r}_1} \left(\frac{\partial}{\partial \mathfrak{p}_1} - \frac{\partial}{\partial \mathfrak{p}_2}\right)$,

$1 \equiv \mathfrak{r}_1\, \mathfrak{p}_1$, $s' \equiv \mathfrak{r}_1\, \mathfrak{p}_s$, the Dirac δ-function and the Fourier transform u_k of the Coulomb potential u_{12}. Thereby

$$\varepsilon_{1,\mathfrak{k}} \equiv 1 - (2\pi)^{3/2} \lim_{\varepsilon \to 0} (-i) \int \mathrm{d}\mathfrak{p}_3 \int_0^\infty \mathrm{d}t' \exp\left\{-i t' \frac{\mathfrak{p}_1 - \mathfrak{p}_3}{m}\, \mathfrak{k} - \varepsilon t'\right\} u_k\, \mathfrak{k}\, \frac{\partial}{\partial \mathfrak{p}_3} n_{3',t} \tag{2}$$

describes a non-stationary screening of the potential.

Transport coefficients for a plasma were calculated by BRAUN 1967 [4] from the LENARD-BALESCU equation using the Chapman-Enskog method. Here the corresponding Kubo formalism will be described. The procedure will be demonstrated at the shear viscosity because of its simplicity. It does not differ essentially for other transport coefficients. The Kubo formula for the shear viscosity is evaluated with the

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[4] E. BRAUN, Phys. Fluids **10**, 731 [1967].

method of ERNST [1] who developed it for dilute gases. It leads to the same integral equation as was derived and further evaluated by BRAUN [4]. Moreover a correction to BRAUN's formula can be given as caused by the internal interaction. It agrees in orders of magnitude with the results of KLIMONTOWITSCH and EBELING [5]. They estimated the correction for the corresponding current density (press tensor) using a corrected LENARD-BALESCU equation. For normal plasmas the correction may be neglected.

In part 2 the Kubo formula for the viscosity is transformed by introducing reduced time correlation functions. Calculating it leads to the same many-body problem which was solved approximately with the derivation of the corresponding kinetic equation. Therefore in part 3 the corresponding linearized kinetic equation for the reduced time correlation function can be used, and it can be transformed into an integral equation.

2. Representation of the Transport Coefficient

The Kubo formula for the shear viscosity η in the homogeneous and isotropic case can be written in the form [1]:

$$\eta = \lim_{\tau \to 0} \frac{1}{10\,k\,T\,V} \langle J^{\alpha\beta} [\exp\{-i\,\tau\,\boldsymbol{l}_{1\dots N}\} - 1]\, M_{\alpha\beta} \rangle\,,$$

$$\boldsymbol{l}_{1\dots N} = \sum_{i}^{1\dots N} \boldsymbol{l}_i + \sum_{i \neq j}^{1\dots N} \boldsymbol{l}_{ij}\,, \tag{3}$$

where the Einstein sum convention for Greek letters is used. (k Boltzmann constant, T temperature, V volume, $\boldsymbol{l}_{1\dots N}$ Liouville operator). $J^{\alpha\beta}$ means a component of the symmetric microscopic momentum current tensor depending on the coordinates and momenta $1\dots N$:

$$\boldsymbol{J} \equiv \sum_{i}^{1\dots N} \frac{\mathfrak{p}_i \circ \mathfrak{p}_i}{m} - \frac{1}{4} \sum_{i \neq j}^{1\dots N} \left(\mathfrak{r}_{ij} \circ \frac{\partial u_{ij}}{\partial \mathfrak{r}_{ij}} + \frac{\partial u_{ij}}{\partial \mathfrak{r}_{ij}} \circ \mathfrak{r}_{ij} \right),$$

$$\mathfrak{r}_{ij} \equiv \mathfrak{r}_i - \mathfrak{r}_j\,. \tag{4}$$

The microscopic moment tensor is defined by

$$\boldsymbol{M} \equiv -\tfrac{1}{2} \sum_{i}^{1\dots N} [\mathfrak{p}_i \circ \mathfrak{r}_i + \mathfrak{r}_i \circ \mathfrak{p}_i - \tfrac{2}{3} \boldsymbol{I} (\mathfrak{r}_i\, \mathfrak{p}_i)]\,. \tag{5}$$

($\boldsymbol{I}$ unit tensor). The brackets $\langle\dots\rangle$ symbolize the equilibrium average with the canonical ensemble:

$$\langle a_{1\dots N} \rangle \equiv \frac{1}{Z^{(N)}} \int \frac{\mathrm{d}1\dots\mathrm{d}N}{N!}\, e^{-\beta\, h_{1\dots N}}\, a_{1\dots N}\,,$$

$$Z^{(N)} \equiv \int \frac{\mathrm{d}1\dots\mathrm{d}N}{N!}\, e^{-\beta\, h_{1\dots N}}\,, \tag{6}$$

$$\beta \equiv \frac{1}{k\,T}\,, \qquad h_{1\dots N} = \sum_{i}^{1\dots N} h_i + \frac{1}{2} \sum_{i \neq j}^{1\dots N} u_{ij}\,.$$

($h_{1\dots N}$ Hamilton function, h_i unperturbed one-particle energy, u_{12} interaction). The other transport coefficients — bulk viscosity, thermal conductivity etc. — differ from (3) only in the factor preceding the average in (3) and in the one- and two-particle quantities in $\boldsymbol{J}$ and $\boldsymbol{M}$. Therefore they can be treated in an analogous way.

Formula (3) can be transformed by introducing reduced time correlation functions

$$\psi^{\alpha\beta}_{1\dots s,\tau} \equiv \frac{1}{Z^{(N)}} \int \frac{\mathrm{d}(s+1)\dots\mathrm{d}N}{(N-s)!} \exp\{-\beta\, h_{1\dots N}\} \exp\{-i\,\tau\,\boldsymbol{l}_{1\dots N}\}\, M^{\alpha\beta} \tag{7}$$

resp. difference functions

$$\Delta\psi^{\alpha\beta}_{1\dots s,\tau} \equiv \psi^{\alpha\beta}_{1\dots s,\tau} - \psi^{\alpha\beta}_{1\dots s,0}\,. \tag{8}$$

Then (3) may be represented by [1]

$$\eta \equiv \eta_{\mathrm{kin}} + \eta_{\mathrm{int}} = \frac{1}{10\,k\,T\,V} \left\{ \int \mathrm{d}1\, \frac{1}{m}\, p_{1,\alpha}\, p_{1,\beta}\, \Delta\psi^{\alpha\beta}_{1,\infty} - \int \frac{\mathrm{d}1\,\mathrm{d}2}{2!}\, \frac{1}{2} \left(r_{12,\alpha} \frac{\partial u_{12}}{\partial r_{12,\beta}} + r_{12,\beta} \frac{\partial u_{12}}{\partial r_{12,\alpha}} \right) \Delta\psi^{\alpha\beta}_{12,\infty} \right\}. \tag{9}$$

The first term in (9) is called kinetic part η_{kin} and the second one interaction part η_{int} relative to their origin.

The reduced correlation functions contain the evolution operator $\exp[-i\,\tau\,\boldsymbol{l}_{1\dots N}]$ which, if applied to the initial distribution function, gives the formal solution of the Liouville equation. Therefore the reduced correlation functions satisfy the BBGKY hierarchy. In order to calculate them one has to solve the many-body problem. This can be done approximately as in [3] by means of perturbation expansion and partial summation. For this purpose a

[5] YU. L. KLIMONTOWITSCH and W. EBELING, Wiss. Veröff. d. Universität Rostock 1962, H. 2, 355.

dimensional analysis of (9) is performed as in [3]. It leads to a double expansion of η in the two system parameters: dilution parameter $\mathcal{N} r_0^3$ ($\mathcal{N}$ mean particle density, r_0 effective range of the interaction) and coupling parameter u_0/kT (u_0 mean effective strength of the interaction):

$$\eta_{\text{kin}} = \mathcal{N} r_0 p_0 \left[\eta_{\text{kin}}^{(0,0)} + \sum_{n \le m}^{1\dots\infty} (\mathcal{N} r_0^3)^n \left(\frac{u_0}{kT}\right)^m \eta_{\text{kin}}^{(n,m)}\right],$$
$$\eta_{\text{int}} = \mathcal{N} r_0 p_0 \sum_{n \le m}^{1\dots\infty} (\mathcal{N} r_0^3)^n \left(\frac{u_0}{kT}\right)^m \eta_{\text{int}}^{(n,m)}. \tag{10}$$

In a plasma we have the Coulomb interaction with infinite range. Therefore even for dilute plasmas the parameter $\mathcal{N} r_0^3$ with the effective range r_0 will be large. The coupling parameter is small in many cases. If one chooses

$$\frac{u_0}{kT} \ll 1, \quad \mathcal{N} r_0^3 \gg 1, \quad \mathcal{N} r_0^3 \frac{u_0}{kT} \approx 1 \tag{11}$$

as a condition for the partial summation, one has taken as mean range the Debye length $r_0 \approx \sqrt{kT/\mathcal{N}\,\epsilon^2}$ because of $u_0 = \epsilon^2/r_0$ and $\mathcal{N} r_0^3 \cdot u_0/kT \approx 1$. Indeed the Debye length appears in the plasma by collective effects. Because of (11) one has to take into account all terms of (10) with $(\mathcal{N} r_0^3 \cdot u_0/kT)^n$ and $(u_0/kT)(\mathcal{N} r_0^3 \cdot u_0/kT)^n$ for all $n = 1, 2, \dots$. Thus we obtain in the Debye-Hückel approximation:

$$\eta_{\text{kin}} \approx \mathcal{N} r_0 p_0 \left[\eta_{\text{kin}}^{(0,0)} + \eta_{\text{kin}}^{(0)} + \frac{u_0}{kT} \eta_{\text{kin}}^{(1)}\right],$$
$$\eta_{\text{kin}}^{(s)} \equiv \sum_n \left(\mathcal{N} r_0^3 \frac{u_0}{kT}\right)^n \eta_{\text{kin}}^{(n,n+s)},$$
$$\eta_{\text{int}} \approx \mathcal{N} r_0 p_0 \left[\eta_{\text{int}}^{(0)} + \frac{u_0}{kT} \eta_{\text{int}}^{(1)}\right], \tag{12}$$
$$\eta_{\text{int}}^{(s)} \equiv \sum_n \left(\mathcal{N} r_0^3 \frac{u_0}{kT}\right)^n \eta_{\text{int}}^{(n,n+s)}.$$

The $\eta^{(0,0)}$ and $\eta^{(s)}$ are dimensionless and of order one. According to this approximation the reduced correlation functions have to be calculated.

3. Approximate Calculation of the Reduced Correlation Functions

As the reduced correlation functions satisfy the BBGKY hierarchy and as the same partial summation under the conditions (11) has already been performed in the case of kinetic equations [3], following ERNST [1] one can use the corresponding kinetic equation — here the LENARD-BALESCU equation — for the one-particle correlation function $\psi_{1,\tau}$:

$$i \frac{\partial}{\partial \tau} \psi_{1,\tau}^{\alpha\beta} = \boldsymbol{l}_1 \psi_{1,\tau}^{\alpha\beta} + \int d2\, \boldsymbol{l}_{12} (\psi_{1,\tau}^{\alpha\beta} n_2^{(0)} + n_1^{(0)} \psi_{2,\tau}^{\alpha\beta}) + \boldsymbol{A} \psi_{1,\tau}^{\alpha\beta}, \tag{13}$$

which is linearized near equilibrium relative to small deviations from

$$n_s^{(0)} \equiv \mathcal{N} [2\pi m k T]^{-3/2} \exp\left\{-\frac{1}{2mkT} \mathfrak{p}_s^2\right\}. \tag{14}$$

The second term on the right-hand side of (13) is the Vlasov term. Because of the coordinate independence of $\Delta\psi_{1,\tau} = \psi_{1,\tau} + \psi_{1,0}$ and the form of $\psi_{1,0}$ (19) it is equal to zero as can be shown by partial integration relative to $\mathfrak{r}_2$.

In (13) the linear integral operator

$$\boldsymbol{A} \psi_{1,\tau}^{\alpha\beta} \equiv \int d2\, \boldsymbol{l}_{12} \boldsymbol{a}_{12}^{(0)} (\psi_{1,\tau}^{\alpha\beta} n_2^{(0)} + \psi_{2',\tau}^{\alpha\beta} n_1^{(0)}) \tag{15}$$

is introduced. The collision operator $\boldsymbol{a}_{12}^{(0)}$ is in the case of the LENARD-BALESCU equation

$$\boldsymbol{a}_{12}^{(0)} \equiv -\int \frac{d\mathfrak{k}}{(2\pi)^{3/2}} \exp\{-i\,\mathfrak{k}\,\mathfrak{r}_{12}\} \cdot \frac{1}{|\varepsilon_{1,\mathfrak{k}}^{(0)}|^2} \delta\left(\frac{\mathfrak{p}_1 - \mathfrak{p}_2}{m} \mathfrak{k}\right) u_k\, \mathfrak{k} \left(\frac{\partial}{\partial \mathfrak{p}_1} - \frac{\partial}{\partial \mathfrak{p}_2}\right), \tag{16}$$

where $\varepsilon_{1,\mathfrak{k}}^{(0)} \equiv \varepsilon_{1,\mathfrak{k}}[n_s^{(0)}]$ means $\varepsilon_{1,\mathfrak{k}}^{(0)}$ depending only on the Maxwell distribution (14). In this form the linearized LENARD-BALESCU equation was also used by BRAUN [4], but for the one-particle distribution $n_{1,t}$.

Because of the coordinate independence of $\Delta\psi_{1,\tau}$ and because of the time independence of $\psi_{1,0}$

$$\boldsymbol{l}_1 \psi_{1,\tau}^{\alpha\beta} = \boldsymbol{l}_1 \psi_{1,0}^{\alpha\beta}, \qquad \frac{\partial}{\partial \tau} \psi_{1,\tau}^{\alpha\beta} = \frac{\partial}{\partial \tau} \Delta\psi_{1,\tau}^{\alpha\beta} \tag{17}$$

are valid. Then (13) becomes an equation for $\Delta\psi_{1,\tau} = \psi_{1,\tau} + \psi_{1,0}$

$$i \frac{\partial}{\partial \tau} \Delta\psi_{1,\tau}^{\alpha\beta} = \boldsymbol{l}_1 \psi_{1,0}^{\alpha\beta} + \boldsymbol{A} \Delta\psi_{1,\tau}^{\alpha\beta} + \boldsymbol{A} \psi_{1,0}^{\alpha\beta}, \tag{18}$$

where

$$\psi_{1,0}^{\alpha\beta} = -\tfrac{1}{2} [r_{1,\alpha} p_{1,\beta} + r_{1,\beta} p_{1,\alpha} - \tfrac{2}{3} \delta_{\alpha\beta} p_{1,}^{\gamma} r_{1,\gamma}]\, n_1^{(0)} \tag{19}$$

is a known function [vid. (7)].

In (9) only $\psi_{1,\infty}$ is needed; therefore in (13) the limit $\tau \to \infty$ has to be taken. Then

$$\lim_{\tau \to \infty} \frac{\partial}{\partial \tau} \Delta\psi_{1,\tau}^{\alpha\beta} = 0 \tag{20}$$

has to be valid in order to get an constant expression for $\Delta\psi_{1,\infty}$ and (9). This is true if the kinetic equation is irreversible such as the LENARD-BALESCU equation. Then (18) becomes an integral equation:

$$\boldsymbol{A}\,\Delta\psi_{1,\infty}^{\alpha\beta} = -\boldsymbol{l}_1\,\psi_{1,0}^{\alpha\beta} - \boldsymbol{A}\,\psi_{1,0}^{\alpha\beta}. \qquad (21)$$

Following ERNST one has now to expand the right-hand side of (21) corresponding to (12) in order to get an expansion of $\Delta\psi_{1,\infty}$ and thereby (12) for η. In the zeroth approximation for $\eta_{\text{kin}}^{(0,0)}$ one has to neglect $\boldsymbol{A}\,\psi_{1,0}$ in (21), because it is of the order $(u_0/k\,T)\,(\mathcal{N}\,r_0{}^3\cdot u_0/k\,T)^n$. The term $\eta^{(0)}$ in (12) can stem only from the Vlasov term, which vanishes. Therefore $\eta^{(0)}$ is zero. Moreover the term $\boldsymbol{A}\,\psi_{1,0}$ in (21) is zero, because it contains

$$\delta\left(\frac{\mathfrak{p}_1-\mathfrak{p}_2}{m}\,\mathfrak{k}\right)\mathfrak{k}\left(\frac{\partial}{\partial\mathfrak{p}_1}-\frac{\partial}{\partial\mathfrak{p}_2}\right)(\psi_{1,0}^{\alpha\beta}\,n_2^{(0)}+n_1^{(0)}\,\psi_{2',0}^{\alpha\beta})$$
$$= -\delta\left(\frac{\mathfrak{p}_1-\mathfrak{p}_2}{m}\,\mathfrak{k}\right)\mathfrak{k}\left(\frac{\partial}{\partial\mathfrak{p}_1}-\frac{\partial}{\partial\mathfrak{p}_2}\right)\frac{1}{2}\,[\,(p_{1,}{}^{\alpha}+p_{2,}{}^{\alpha})\,r_{1,}{}^{\beta}(p_{1,}{}^{\beta}+p_{2,}{}^{\beta})\,r_{1,}{}^{\alpha}-\tfrac{2}{3}\delta^{\alpha\beta}(p_{1,}{}^{\gamma}+p_{2,}{}^{\gamma})\,]\,r_{1,\gamma}\,n_1^{(0)}n_2^{(0)}=0\,. \qquad (22)$$

In the Debye-Hückel approximation the kinetic part of η thus contains only $\eta_{\text{kin}}^{(0,0)}$, where $\Delta\psi_{1,\infty}$ can be obtained from the integral equation

$$\boldsymbol{A}\,\Delta\psi_{1,\infty}^{\alpha\beta} = -i\,\frac{1}{m}\,[p_{1,}{}^{\alpha}\,p_{1,}{}^{\beta}-\tfrac{1}{3}\,\delta^{\alpha\beta}\,p_{1,}{}^{\gamma}\,p_{1,\gamma}]\,n_1{}^{(0)} \qquad (23)$$

after applying $\boldsymbol{l}_1$ to $\psi_{1,0}$. This integral equation has been found and solved by BRAUN [1]. For reasons of symmetry he made the ansatz

$$\Delta\psi_{1,\infty}^{\alpha\beta} \equiv \frac{1}{m}\,[p_{1,}{}^{\alpha}\,p_{1,}{}^{\beta}-\tfrac{1}{3}\,\delta^{\alpha\beta}\,p_{1,}{}^{\gamma}\,p_{1,\gamma}]\,B(|\,\mathfrak{p}_1|)\,n_1{}^{(0)}, \qquad (24)$$

so that only the scalar function $B(|\,\mathfrak{p}_1|)$ from (23) with (24) is necessary for the calculation of $\Delta\psi_{1,\infty}$. Now the comparison of the first BBGKY equation

$$i\,\frac{\partial}{\partial\tau}\,\psi_{1,\tau}^{\alpha\beta} = \boldsymbol{l}_1\,\psi_{1,\tau}^{\alpha\beta} + \int \mathrm{d}2\;\boldsymbol{l}_{12}\,\psi_{12,\tau}^{\alpha\beta} \qquad (25)$$

with (13) shows that in this approximation

$$\psi_{12,\tau}^{\alpha\beta} = \boldsymbol{a}_{12}^{(0)}\,(\psi_{1,\tau}^{\alpha\beta}\,n_2{}^{(0)}+\psi_{2',\tau}^{\alpha\beta}\,n_1{}^{(0)}) \qquad (26)$$

is valid. Because of symmetry $\psi_{12,0}$ does not contribute to (9), and because of $\boldsymbol{A}\,\psi_{1,0}^{\alpha\beta}=0$ one gets

$$\Delta\psi_{12,0}^{\alpha\beta} = \boldsymbol{a}_{12}^{(0)}\,(\Delta\psi_{1,\infty}^{\alpha\beta}\,n_2^{(0)}+\Delta\psi_{2',\infty}^{\alpha\beta}\,n_1^{(0)}) \qquad (27)$$

with $\boldsymbol{a}_{12}^{(0)}$ from (16).

By way of summarizing one obtains for the shear viscosity of a Debye-Hückel plasma

$$\begin{aligned}\eta_{\text{kin}} &= \frac{1}{15\,m^2\,k\,T}\int \mathrm{d}\mathfrak{p}_1\,\mathfrak{p}_1{}^4\,B(|\,\mathfrak{p}_1|)\,n_1^{(0)},\\ \eta_{\text{int}} &= -\frac{1}{10\,k\,T\,V}\int\frac{\mathrm{d}1\,\mathrm{d}2}{2!}\,\frac{1}{2}\left(r_{12,\alpha}\frac{\partial u_{12}}{\partial r_{12,\beta}}+r_{12,\beta}\frac{\partial u_{12}}{\partial r_{12,\alpha}}\right)\boldsymbol{a}_{12}^{(0)}\\ &\quad\times n_1^{(0)}\,n_2^{(0)}\sum_i^{1,2}\frac{1}{m}\,[p_{1,}{}^{\alpha}\,p_{1,}{}^{\beta}-\delta^{\alpha\beta}\,\tfrac{1}{3}\,p_{1,}{}^{\gamma}\,p_{1,\gamma}]\,B(|\,\mathfrak{p}_i|)\,,\end{aligned} \qquad (28)$$

where only $B(|\,\mathfrak{p}_i|)$ has to be calculated from (23) with (24). This has been done by BRAUN [4], who found only the term η_{kin}. As in (12) the first term for η_{int} stems from the Vlasov term and therefore vanishes, the estimation

$$\frac{\eta_{\text{int}}}{\eta_{\text{kin}}} \approx \frac{u_0}{k\,T} = \frac{r_{\mathrm{L}}}{r_{\mathrm{D}}}, \qquad r_{\mathrm{L}} \equiv \frac{\epsilon^2}{k\,T}, \qquad r_{\mathrm{D}} \equiv \sqrt{\frac{\mathcal{N}\epsilon}{k\,T}} \qquad (29)$$

can be obtained from (12). It is in agreement with the results of [5]. For a typical plasma the plasma parameter is $u_0/k\,T = r_{\mathrm{L}}/r_{\mathrm{D}} \approx 10^{-6}$ and the interaction part may be neglected.

So the Kubo formalism leads to the same results as derived from the kinetic equation. Moreover formula (28) makes it possible to exactly calculate the interaction part of the viscosity.